Complete Bell state measurement with controlled photon absorption and quantum interference

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A solid state device to discriminate all the four Bell states is proposed. The device is composed of controlled absorption crystals, rotators, and retarders. The controlled absorption, where the state of one photon affects the absorption of the other photon, is realized by two photon absorption in a cubic crystal. The controlled absorption crystal detects a particular Bell state and is transparent for the other Bell states. The rotators and retarders transform a Bell state to another. This device may solve the problems in the early quantum teleportation experiments in photon polarization states.

The quantum information technology aims to achieve performance in communication and computation systems superior to those based on classical physics by utilizing the entanglement between particles. Bell states, the maximally entangled two-particle states, are crucial ingredients in the quantum information technology, such as quantum teleportation [1], dense coding [2], and entanglement swapping [3]. The quantum teleportation transfers unknown quantum states. Dense coding offers larger capacity than the classical communication. The entanglement swapping creates entanglement of two particles that have never interacted, and provides multi-particle entanglement. Quantum repeaters [4], which are indispensable elements in quantum networks to relay unknown quantum states in a long distance, can be realized with the quantum teleportation and the entanglement swapping. The key devices for the above quantum information processing are generators and discriminators of the four Bell states:

$$\begin{aligned} \left|\Phi^{(\pm)}\right\rangle &= \frac{1}{\sqrt{2}}\left(|x\rangle_1 |x\rangle_2 \pm |y\rangle_1 |y\rangle_2\right) \\ \left|\Psi^{(\pm)}\right\rangle &= \frac{1}{\sqrt{2}}\left(|x\rangle_1 |y\rangle_2 \pm |y\rangle_1 |x\rangle_2\right). \end{aligned} \tag{1}$$

A parametric downconversion in a nonlinear crystal generates one of the two-photon Bell states. The other three Bell states are easily obtained by suitable unitary operations with linear polarization elements (such as retarders and rotators [5].) On the other hand, a realizable complete Bell state measurement (BSM) method, which discriminates all the four Bell states, has been little known. Lütkenhaus *et al* [6] have proved a no-go theorem that shows the impossibility of never failing BSM with linear optical elements and detectors. The lack of the complete BSM limits the universality of the quantum teleportation experiments. Bouwmeester *et al* [7] has succeeded the quantum teleportation on the polarization states in only 25 % of the input states, because they discriminated only one ($\Psi^{(-)}$) state out of the four Bell states. Boschi *et al* [8] reported 100 % efficiency with linear optical elements by implementing the Bell states on the product space of the two degrees freedom of the same system. The teleported state had to be prepared in the system and was known before the teleportation. A controlled NOT gate transforms the Bell states into disentangled states to be easily discriminated, and provides a quantum circuit for the complete BSM [9]. However, a controlled NOT gate itself is beyond current technology.

Recently, a progress towards a realizable BSM has been made. Scully *et al* [10] proposed a two-photon absorption (TPA) scheme for the complete BSM. Unlike the quantum gates, coherency is not required in the output of the TPA detection scheme; the measurement is done in the TPA process. Therefore, the photon energy can be resonant to the atomic two-photon transition. This resonance may enhance the TPA to resolve the low efficiency problem of the nonlinear crystals [2]. The TPA scheme [10], however, requires three gas cells filled with atoms prepared in special coherent superpositions of hyperfine states to discriminate the Bell states. It is still not easy to prepare the coherent superposition; it requires other strong laser sources with carefully tuned frequencies and phases. In this letter we propose more realizable devices for the complete BSM. We will show that a particular Bell state can be discriminated without any state preparation, and that the discrimination of only one Bell state is enough for the complete BSM with the help of linear polarization elements that transform one Bell state to another.

The complete BSM can be achieved as follows. We found that a controlled absorption and quantum interference result in the discrimination of a particular Bell state. The "controlled absorption" refers to the phenomena that the state of one photon affects the absorption of the other photon. An example of the controlled absorption is TPA in crystals. The TPA coefficient in a crystal depends on the polarizations of the two photons (two-photon polarization

selection rule.) The quantum interference will take place if the transition to the same final state is possible through two routes. The following example illustrates the Bell state discrimination. Suppose two photons in one of the four Bell state enter a crystal. The Bell states are given by Eq.(1) in the linear polarization bases: $|x\rangle$ ($|y\rangle$) denotes the photon polarized in x (y) direction, as in the Innsbruck experiments [7]. If the two-photon polarization selection rule in the crystal allows the TPA with the photons only in the same polarizations, i.e., $|x\rangle_1 |x\rangle_2$ and $|y\rangle_1 |y\rangle_2$, absorption of the photons in the $\Psi^{(\pm)}$ states is forbidden. Moreover, if the two combinations of photons $|x\rangle_1 |x\rangle_2$ and $|y\rangle_1 |y\rangle_2$ contribute to the transition to the same final state with the same amount, the quantum interference between the transitions by $|x\rangle_1 |x\rangle_2$ and $|y\rangle_1 |y\rangle_2$ photons enhances the TPA of the $\Phi^{(+)}$state but suppresses that of the $\Phi^{(-)}$ state. Detection of TPA by two-photon luminescence or two-photon photoconductivity will thus indicate that $\Phi^{(+)}$ state entered the crystal. The TPA from the Γ_1^+ ground state to the Γ_1^+ final state in a cubic crystal obeys the above two-photon polarization selection rule and shows the quantum interference, as shown later. The crystal is transparent to the photons in the other Bell states. The other Bell states can be discriminated by the crystal after the state transformation to the $\Phi^{(+)}$ state. A $\pi/2$-retarders (quarter-wave plates) applied to both photons produce a phase difference $\pi/2$ between x-polarized photons and y-polarized photons [11], and thus transform the $\Phi^{(-)}$ state to the $\Phi^{(+)}$ state, but leave the $\Psi^{(\pm)}$ states unchanged except a common phase factor. Rotating the polarization of one photon by $\pi/2$ interchanges the $\Psi^{(\pm)}$ states and the $\Phi^{(\mp)}$ states. 1 depicts a complete BSM device composed of the TPA crystals, linear polarization elements, and detectors. The crystals absorb only the photons in the $\Phi^{(+)}$ state. The $\Phi^{(+)}$ state is discriminated by the first crystal. The transmitted states are transformed by the $\pi/2$-retarders. Then the second crystal discriminates the $\Phi^{(-)}$ state. The photons transmitted the second crystal must be in the $\Psi^{(\pm)}$ states, which are transformed to the $\Phi^{(\mp)}$ states by the rotator. The third crystal thus discriminates the $\Psi^{(-)}$ state. Putting $\pi/2$-retarders to the transmitted beams, we can detect the $\Psi^{(+)}$ state by the forth crystal . All the four Bell states are successfully discriminated. If the TPA crystal discriminates another Bell state by different two-photon polarization selection rule, complete BSM can be achieved similarly but with a different combination of retarders and rotators. It seems that the complete BSM may be possible with a device composed of three TPA crystals that discriminate three Bell states, and an ordinary photo-detector that detects the photon transmission from the three TPA crystals. If the photons are detected in the photo-detector, one may conclude the photons are in the forth Bell state. It is not always true, however, because the TPA probability is less than unity. The photo-detector would detect photons that the TPA crystals failed to absorb. The failure in TPA discrimination will results in the error in BSM.

The two-photon polarization selection rule is derived from the second order perturbation theory [12]. We will consider TPA in a cubic crystal belonging to the point group O_h, for simplicity. The crystal ground state is assumed to belong to the Γ_1^+ irreducible representation of energy E_0, the final state $|f_m^\mu\rangle$ to the m-th row of the irreducible representation D^μ of energy E_{f^μ}. The total momentum vector $\mathbf{P}$ belongs to the irreducible representation $D^\lambda = \Gamma_4^-$, and the final state should belong to one of the decomposition of the product representations as $\Gamma_4^- \otimes \Gamma_4^- = \Gamma_1^+ \oplus \Gamma_3^+ \oplus \Gamma_4^+ \oplus \Gamma_5^+$. The Wigner-Eckart theorem separate the reduced matrix elements and the geometrical factor of the TPA rate of two photons with the energies $\hbar\omega_1$ and $\hbar\omega_2$ [13] [14]:

$$\alpha\left(\omega_1,\omega_2\right) \propto \sum_{f^\mu} \left| \sum_m G_{\mu m}\left(\psi\right) \sum_{\varphi^\lambda} \Lambda^{\pm}_{\varphi^\lambda} \left\langle f^\mu \left\| P^\lambda \right\| \varphi^\lambda \right\rangle \left\langle \varphi^\lambda \left\| P^\lambda \right\| 0 \right\rangle \right|^2 \delta\left(E_{f^\mu} - E_0 - \hbar\omega_1 - \hbar\omega_2\right), \tag{2}$$

where the energy denominator is defined by

$$\Lambda^{\pm}_{\varphi^\lambda} = \frac{1}{E_{\varphi^\lambda} - E_0 - \hbar\omega_1} \pm \frac{1}{E_{\varphi^\lambda} - E_0 - \hbar\omega_2}. \tag{3}$$

The positive (negative) sign in the energy denominator (3) holds for the final states belonging to the symmetric (antisymmetric) decomposition of $D^\lambda \otimes D^\lambda$. TPA to the states belonging to the antisymmetric decomposition (Γ_4^+) is weak for nearly degenerate photons $\omega_1 \sim \omega_2$. We will focus the final states belonging to the symmetric decomposition (Γ_1^+, Γ_3^+, and Γ_5^+). The two-photon polarization selection rule is determined by the geometrical factor

$$G_{\mu m}\left(\psi\right) = \sum_{ll'} \left\langle vac\right| a_1^l a_2^{l'} \left|\psi\right\rangle \left(\mu m | \lambda l, \lambda l'\right), \tag{4}$$

where $(\mu m|\lambda l, \lambda l')$ refers to the Clebsh-Gordan coefficients. We have generalized the expression by Doni *et al* [14] to include the non-classical photon states $|\psi\rangle$. The operator $a_1^l (a_2^{l'})$ is the $l(l')$-th raw of the annihilation operators of photon 1(2). The Clebsh-Gordan coefficients for the symmetric decomposition of the product representation can be found in Ref. [15]. If the final state belongs to the Γ_1^+ irreducible representation, the Clebsh-Gordan coefficients read $\left(\Gamma_1^+|\Gamma_4^- l, \Gamma_4^- l'\right) = \left(1/\sqrt{3}\right)\delta_{l,l'}$. The geometrical factor

$$G_{\Gamma_1^+}(\psi) = \frac{1}{\sqrt{3}} \langle vac| a_1^x a_2^x |\psi\rangle + \frac{1}{\sqrt{3}} \langle vac| a_1^y a_2^y |\psi\rangle + \frac{1}{\sqrt{3}} \langle vac| a_1^z a_2^z |\psi\rangle \tag{5}$$

takes $2/\sqrt{3}$ for the $\Phi^{(+)}$ state and 0 for the $\Phi^{(-)}$ state and the $\Psi^{(\pm)}$ states, i.e., TPA occurs for only $\left|\Phi^{(+)}\right\rangle$ state in $\Gamma_1^+ \to \Gamma_1^+$ transition. TPA is allowed for two photons in parallel polarization in this transition. The interference between the TPA process of xx-polarized photons and yy-polarized photons cancels the geometrical factor for the $\Phi^{(-)}$ state. The selection rules for the transition to the final states belonging to other irreducible representations can be derived similarly. The same selection rules hold to the crystals belonging to other cubic symmetry groups like T_d.

The Bell state detection requires a large TPA coefficient β. A promising candidate is the two-photon transition to the biexciton states. The giant oscillator strength and the resonance of the intermediate state (exciton states) result in huge enhancement in TPA coefficient, which was estimated to be 10^7 in CuCl [16]. The biexcitons in CuCl belongs to the Γ_1 irreducible representation of the T_d point group. The TPA photon energy lies at 3.186 eV, well separated from the exciton transition at 3.202 eV to avoid undesirable one-photon absorption. A cavity will enhance the TPA coefficient to improve the detection efficiency. We assume that two-photons of frequency ω are confined in a cavity of volume V filled by a CuCl crystal. The electric field in the cavity is given by $E = \left(\hbar\omega/n^2\epsilon_0 V\right)^{1/2}$, where n is the refractive index in the cavity and ϵ_0 is the permittivity of the vacuum. The TPA rate α in the cavity reads

$$\alpha = \left(\frac{c}{n}\right)^2 \epsilon_0 \beta E^2 = \frac{c^2\beta\hbar\omega}{n^4 V}. \tag{6}$$

Putting the values $\hbar\omega = 3.186$ eV, $n \sim 3$, $\beta \sim 0.1$ cm/W, and $V \sim 1$ μm^3, we obtain the value of the TPA rate: $\alpha \sim 6 \times 10^{11}$ s^{-1}. The condition to obtain efficient TPA is that the photon life time of the cavity is longer than $\alpha^{-1} \sim 1.7$ ps. This implies the Q value of the cavity to be larger than $8{\times}10^3$, which can be satisfied by the current technology. The complete BSM by a solid state device is thus be realizable in the present TPA detection scheme.

BSM with TPA has a further advantage. We can set the photon energy of one beam to be different from that of the other beam, and the sum of the energies to be resonant to the two photon transition, i.e., $\hbar\omega_1 \neq \hbar\omega_2$ and $\hbar\omega_1 + \hbar\omega_2 = E_{f^\mu} - E_0$. This nondegenerate BSM can rule out the possibility that detects the two photons from one source, because the non-resonant TPA of the photons with the same energy will be weak. It will remove the difficulty in the Innsbruck experiments [7] pointed out by Braunstein and Kimble [17].

The controlled absorption can be realized in other systems. We here consider the optical transition between the lowest sublevels in a quantum dot. We assume the lowest hole states are the heavy hole states $|3/2, \pm 3/2\rangle_h$. Then the electron-hole pair states result from the optical transition are $|\!\uparrow\rangle = |1/2, -1/2\rangle_e |3/2, -3/2\rangle_h$ and $|\!\downarrow\rangle = |1/2, 1/2\rangle_e |3/2, 3/2\rangle_h$; the former is created by a right-handed circularly polarized photons $|\sigma^+\rangle$ and the latter by a left-handed circularly polarized photons $|\sigma^-\rangle$. Suppose one electron-hole pair exists in the quantum dot, and define Bell states of an electron-hole pair and a photon:

$$\begin{aligned} \left|\Phi^{(\pm)}\right\rangle &= \frac{1}{\sqrt{2}} \left(|\!\uparrow\rangle |\sigma^+\rangle \pm |\!\downarrow\rangle |\sigma^-\rangle\right) \\ \left|\Psi^{(\pm)}\right\rangle &= \frac{1}{\sqrt{2}} \left(|\!\uparrow\rangle |\sigma^-\rangle \pm |\!\downarrow\rangle |\sigma^+\rangle\right) \end{aligned} \tag{7}$$

These states are created when the quantum dot absorbs one of the two photons in the Bell states (Eq. 1.) The state of two electron-hole pairs should be in the form $\left(1/\sqrt{2}\right)\left(|\!\uparrow\rangle_1 |\!\downarrow\rangle_2 + |\!\downarrow\rangle_1 |\!\uparrow\rangle_2\right)$, so that only the $\Psi^{(+)}$ state in Eq. 7 is absorbed by the quantum dot. This controlled absorption results from Pauli exclusion principle. Linear polarization elements also transform the Bell states. A π-retarder, which transforms the $|\sigma^+\rangle$ polarization state to the $|\sigma^-\rangle$ state, interchanges the $\Phi^{(\pm)}$ states and the $\Psi^{(\pm)}$ states. A $\pi/2$-rotator, which provides relative phase (-1) between the $|\sigma^+\rangle$ polarization state and the $|\sigma^-\rangle$ state, exchange the signs as $\Phi^{(\pm)} \to \Phi^{(\mp)}$ and $\Psi^{(\pm)} \to \Psi^{(\mp)}$. The light beam should go through the quantum dot four times, because the electron stays in the excited quantum dot. The states are discriminated by the time of the photon detection event. Therefore, the electron-hole state should remain until the Bell state discrimination is completed. The time for the Bell discrimination would be determined by the time resolution of the photon detection. This requirement may limit the feasibility of the quantum dot BSM devices.

The controlled absorption would be useful for other quantum circuit than the complete BSM. The TPA scheme can be regarded as an integrated device of a two-qubit quantum gate and a detector. It will provide a quantum circuit where the qubits pass through quantum gates only once. Multi-photon absorption process may realize more complicated quantum circuits, instead of cascading discrete quantum gates.

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FIG. 1. A proposed device for the complete Bell state measurement composed of two-photon absorbing crystals (X), $\pi/2$-retarders (quater-wave plates), and $\pi/2$-rotators. The crystals absorb the $\Phi^{(+)}$ state, and the absorption is detected. The retaders and the rotators transform one Bell state to another.

Fig.1 A. Tomita, “Complete Bell state measurement...”, Phys. Rev. Lett.